

Fully nonlinear equivalence of δN and covariant formalisms

Teruaki Suyama¹, Yuki Watanabe² and Masahide Yamaguchi³

¹ *Research Center for the Early Universe (RESCEU), Graduate School of Science,
The University of Tokyo, Tokyo 113-0033, Japan*

² *Arnold Sommerfeld Center for Theoretical Physics,
Ludwig Maximilian University of Munich, Theresienstrasse 37, 80333 Munich, Germany*

³ *Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

Abstract

We explicitly show the fully nonlinear equivalence of the δN and the covariant formalisms for the superhorizon curvature perturbations, which enables us to safely evaluate the non-Gaussian quantities of the curvature perturbation in either formalism. We also discuss isocurvature perturbations in the covariant formalism and clarify the relation between the fully nonlinear evolution of the curvature covector and that of the curvature perturbation for multiple interacting fluids.

1 Introduction

Inflation is now widely accepted as the mechanism of generating primordial density fluctuations, which are almost scale invariant, adiabatic, and Gaussian. Such fluctuations are generated inside the horizon and expanded to superhorizon scales due to the inflationary expansion of the Universe. These features are confirmed by the recent observations of cosmic microwave background anisotropies like WMAP experiments [1]. In particular, the large-scale anticorrelation of the temperature and the E-mode polarization strongly supports the presence of the superhorizon epochs of primordial curvature perturbations [2]. Thus, the analysis of the superhorizon evolution of the curvature perturbations is extremely important.

Conventionally, the standard perturbative approach has been adopted to investigate the curvature perturbations [3]. Linear analysis gives simple equations and solutions, which are used to evaluate the power spectrum of the curvature perturbations. However, recent observations are precise enough to probe the nonlinear effects such as the bispectrum and the trispectrum of the curvature perturbations, which are crucially important to identify the origin of the primordial density fluctuations. Though all of the single-field inflation models with the canonical kinetic term predict negligible nonlinear effects on the curvature perturbations [4] (see also [5] for a single-field high friction model), inflation models with noncanonical kinetic terms [6] and light field models [7] such as curvaton [8, 9] and modulated reheating [10, 11] can predict large non-Gaussianity of the curvature perturbations. Therefore, the standard perturbative approach must be extended to deal with the nonlinear effects. But the equations and their solutions often become too complicated to be understood intuitively. Then, instead of the conventional perturbative approach, two different formalisms are proposed to deal with the fully nonlinear curvature perturbations. One is the δN formalism [12] and the other is the covariant formalism [13, 14]^{#1}.

The δN formalism provides a powerful and simple method to investigate the superhorizon evolution of the fully nonlinear curvature perturbations. According to the δN formalism, the superhorizon curvature perturbation on a uniform energy density hypersurface at a late time is equal to the perturbation in the time integral of the local expansion from an initial flat hypersurface to a final uniform energy density hypersurface. At leading order in gradient expansion, which corresponds to the separate Universe approach [16], each superhorizon sized region of the Universe evolves in time independently from other regions. Thus, we have only to evaluate the expansion of the unperturbed Friedmann Universe. In fact, it is quite easy, by the use of the δN formalism, to show that the fully nonlinear curvature perturbation on a uniform energy density hypersurface is conserved on superhorizon scales if the pressure is only a function of the energy density [17].

On the other hand, the covariant formalism defines covariant quantities corresponding to physical quantities and derives their evolution equations. Since these quantities are

^{#1}The space gradient of the fully nonlinear curvature perturbations was first introduced in [15] which is based on the leading order of the coordinate-based gradient expansion. It was later extended to the curvature covector in a covariant manner by [13, 14].

tensor, they are independent of a particular selection of a coordinate system and easy to understand from a geometrical perspective. In particular, the fully nonlinear evolution equations of the curvature covectors corresponding to the curvature perturbations are easily derived. Notice that, different from the δN formalism, they are exact and valid at all scales. Its covector is shown to be conserved for adiabatic perturbations [13].

Then, one may wonder what is the relation between the above two formalisms. In particular, what is the relation between the curvature perturbation and the curvature covector? At the linear, second and third orders [13, 18, 19],^{#2} the curvature covector is shown to be related to the gradient of the curvature perturbations of the standard perturbation theory [20]. For the case of a barotropic fluid, the relation at the fully nonlinear order has been discussed in [21]. One of the present author (Y.W.) has evaluated the bispectrum of the curvature perturbations in two-field inflation models by two different formalisms and explicitly shown that both of them coincide [22]. Similar analyses in ekpyrotic models for the bispectrum [23] and the trispectrum [19] have been done, but they have less accurate quantitative agreement than [22]. Nevertheless, to some extent, the δN formalism and the covariant formalism is shown to be equivalent for superhorizon curvature perturbations up to third orders. In this paper, we explicitly show the fully nonlinear equivalence of the δN and the covariant formalisms for the superhorizon curvature perturbations.

The organization of this paper is as follows. In the next two sections, we briefly review the δN and the covariant formalisms, and derive the evolution equations for the curvature perturbation and the curvature covector from the continuity equation. In Sec. 4, we show the fully nonlinear equivalence of the two formalisms for the superhorizon curvature perturbations. In Sec. 5, isocurvature perturbations in the covariant formalism are discussed. We clarify the relation between the fully nonlinear evolution of the curvature covector and that of the curvature perturbation for multiple interacting fluids. The final section is devoted to the summary.

2 δN formalism

In this section, we briefly review the fully nonlinear version of the δN formalism, picking up the relevant points derived in [17].

Let us start from the Arnowitt-Deser-Misner (ADM) decomposition of the metric,

$$ds^2 = -\mathcal{N}^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt). \quad (1)$$

We further decompose γ_{ij} as

$$\gamma_{ij} = a^2(t)e^{2\psi}(e^h)_{ij}, \quad (2)$$

where $a(t)$ is the fiducial scale factor, h_{ij} is a traceless tensor, and ψ is the perturbation of a unit spatial volume.

^{#2}In Ref. [18], they define a scalar quantity similar to the curvature perturbation ζ and discuss its evolution at all orders.

The gradient expansion assumes that all the quantities of interest are smooth over very large scales [24]. Practically, this expansion is achieved by a prescription that a spatial derivative acting on any variable is accompanied by a small quantity ϵ and we expand all the equations in terms of ϵ . It leads to the so-called δN formalism by keeping only terms at zeroth and first orders in ϵ and dropping higher order terms. As in the literature, we assume that ^{#3}

$$\beta^i = \mathcal{O}(\epsilon), \quad h'_{ij} = \mathcal{O}(\epsilon^2), \quad (3)$$

where the prime ($'$) denotes differentiation with respect to the cosmic time t .

Assuming that the anisotropic stress is $\mathcal{O}(\epsilon^2)$, which is valid in many cases, we can write the large-scale energy-momentum tensor of matter filling up the Universe as

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu} + \mathcal{O}(\epsilon^2). \quad (4)$$

The four-vector is normalized to be $u_\mu u^\mu = -1$ and the integration curve along u^μ gives the fluid worldline. Expansion of u^μ along the worldline is given by

$$\Theta = \nabla_\mu u^\mu = \frac{3}{\mathcal{N}} \left(\frac{a'}{a} + \psi' \right) + \mathcal{O}(\epsilon^2). \quad (5)$$

Correspondingly, the local expansion rate \tilde{H} measured by the local observer using his proper time is

$$\tilde{H} = \frac{1}{3}\Theta = \frac{1}{\mathcal{N}} \left(\frac{a'}{a} + \psi' \right) + \mathcal{O}(\epsilon^2). \quad (6)$$

The e-folding number of the expansion rate along the worldline is defined as

$$N(t_2, t_1; x^i) = \frac{1}{3} \int_{t_1}^{t_2} dt \mathcal{N} \Theta = \log \frac{a(t_2)}{a(t_1)} + \psi(t_2, x^i) - \psi(t_1, x^i) + \mathcal{O}(\epsilon^2). \quad (7)$$

Therefore, if the $t = \text{const.}$ hypersurface at initial time t_1 is taken to be a flat one, i. e. $\psi(t_1, x^i) = 0$, this formula states that perturbation of the e-folding number from t_1 to t_2 coincides with the curvature perturbation at $t = t_2$ (up to first order in ϵ). Equation (7) is the essence of the δN formalism.

Time evolution of ψ can be derived from the continuity equation $u^\mu \nabla_\nu T^\nu_\mu = 0$;

$$\frac{a'(t)}{a(t)} + \psi'(t, x^i) = - \frac{\rho'(t, x^i)}{3[\rho(t, x^i) + P(t, x^i)]} + \mathcal{O}(\epsilon^2). \quad (8)$$

For a fluid having a barotropic equation of state $P = P(\rho)$, we can integrate this equation to obtain a conserved quantity ζ given by

$$\zeta(x^i) = \psi(t, x^i) + \frac{1}{3} \int_{\bar{\rho}(t)}^{\rho(t, x^i)} \frac{d\rho}{\rho + P(\rho)}. \quad (9)$$

In particular, if we take a uniform energy density hypersurface Σ_ρ , the second term in the right hand side vanishes. Therefore, the curvature perturbation on that hypersurface remains constant in time.

^{#3}Mathematically speaking, the second assumption is not needed to derive the evolution equation of ψ in Eq. (8) but needed to interpret ψ as the spatial curvature.

3 Covariant formalism

In this section, we briefly review the covariant formalism developed in Refs. [13, 14].

While the δN formalism takes the coordinate-based approach from the outset, the covariant formalism first defines fully nonlinear and covariant quantities and then derives the basic equations. After this, we can choose a coordinate system and expand the quantities and equations up to the desired order.

In the covariant formalism, the curvature perturbation is represented by a covector ζ_a defined by

$$\zeta_a = \partial_a N - \frac{\dot{N}}{\dot{\rho}} \partial_a \rho = \partial_a N - \frac{\Theta}{3\dot{\rho}} \partial_a \rho. \quad (10)$$

We use a, b, \dots to stress that we are working in the covariant approach. Here N is the local e-folding number along the fluid worldline introduced by Eq. (7) up to an integration constant that vanishes on some hypersurface. In the covariant form, it is given by

$$N = \frac{1}{3} \int d\tau \Theta, \quad (11)$$

where τ is a proper time of the fluid. The continuity equation $u^a \nabla_b T_a^b = 0$ becomes

$$\dot{\rho} + \Theta(\rho + P) = 0, \quad (12)$$

where we have assumed that the Universe is filled up with a perfect fluid. Using this equation, we can derive the evolution equation for ζ_a along the fluid world-line;

$$\dot{\zeta}_a = -\frac{\Theta}{3(\rho + P)} \left(\partial_a P - \frac{\dot{P}}{\dot{\rho}} \partial_a \rho \right). \quad (13)$$

Here the dot ($\dot{}$) denotes the Lie derivative along the fluid worldline. For example,

$$\dot{P} = u^a \nabla_a P, \quad \dot{\zeta}_a = u^b \partial_b \zeta_a + \zeta_b \partial_a u^b. \quad (14)$$

It is worth stressing that the evolution equation (13) is exact and valid on all length scales. For fluids having a barotropic equation of state $P = P(\rho)$, the right-hand side of Eq. (13) vanishes and ζ_a is therefore conserved along the worldline.

4 Equivalence between the two approaches

Now, let us again choose a coordinate system given by Eq. (1) with the condition (3) and expand ζ_a on this coordinate. For the time component, we find

$$\zeta_0 = N' + \frac{\rho'(t, x^i)}{3[\rho(t, x^i) + P(t, x^i)]} = \mathcal{O}(\epsilon^2), \quad (15)$$

where we have used the continuity equation (12) to show the last equality. For the spatial component, we have

$$\zeta_i = \partial_i N + \frac{1}{3[\rho(t, x^i) + P(t, x^i)]} \partial_i \rho(t, x^j). \quad (16)$$

Although the right hand side is not generally a total derivative^{#4}, it becomes a total derivative on a uniform energy density hypersurface Σ_ρ . Therefore, we find that ζ_i is the spatial derivative of the perturbation of the e-folding number evaluated on Σ_ρ . Using Eq. (7), ζ_i can be written as

$$\zeta_i(t_2, x^i) = \partial_i(\psi(t_2, x^j) - \psi(t_1, x^j)) + \mathcal{O}(\epsilon^3), \quad (17)$$

where $\psi(t_2, x^j)$ must be understood as the curvature perturbation on Σ_ρ . In particular, if we take the flat hypersurface at $t = t_1$, the above equation reduces to

$$\zeta_i(t_2, x^i) = \partial_i \psi(t_2, x^j) + \mathcal{O}(\epsilon^3). \quad (18)$$

At this stage, it is clear that ζ_i on very large scales is merely a spatial derivative of the curvature perturbation on Σ_ρ .

We can explicitly show that the evolution equation (13) evaluated on Σ_ρ indeed leads to Eq. (8). The left hand side of Eq. (13) is given by

$$\dot{\zeta}_i = \frac{1}{\mathcal{N}} \partial_i \psi' + \mathcal{O}(\epsilon^3). \quad (19)$$

On the other hand, the right hand side is given by

$$-\frac{\Theta}{3(\rho + P)} \left(\partial_i P - \frac{\dot{P}}{\dot{\rho}} \partial_i \rho \right) = -\frac{\tilde{H}(t, x^i)}{\rho(t) + P(t, x^i)} \partial_i P(t, x^j). \quad (20)$$

Then using a formula for \mathcal{N} valid on Σ_ρ ,

$$\mathcal{N} = \frac{\rho(t) + P(t)}{\rho(t) + P(t, x^i)}, \quad (21)$$

^{#4}One exception is for the fluid having a barotropic equation of state $P = P(\rho)$. In [21] it has been shown that

$$\zeta_i = \partial_i \left[N + \frac{\log \rho}{3(1+w)} \right],$$

where $w = P(\rho)/\rho$ is constant in space. Integrating over space and choosing the integration constant properly, one can immediately get

$$\zeta = \delta N + \frac{1}{3} \int_{\bar{\rho}(t)}^{\rho} \frac{d\tilde{\rho}}{(1+w)\tilde{\rho}}, \quad \delta N \equiv N - \bar{N}(t),$$

which is equivalent to Eq. (9). In the following, we do not demand the barotropic condition.

and a fact that Σ_ρ and the uniform Hubble hypersurface coincide to first order in ϵ [17], we find that Eq. (13) reduces to

$$\partial_i \psi' = \partial_i \left[H(t) \frac{\rho(t) + P(t)}{\rho(t) + P(t, x^j)} \right] + \mathcal{O}(\epsilon^3), \quad (22)$$

whose integration over x^i , combined with the continuity equation $\rho'(t)/\mathcal{N} = -3H(t) [\rho(t) + P(t, x^i)]$, coincides with Eq. (8) provided the integration constant is chosen properly, i.e., a'/a . The curvature perturbation associated with its covector obeys the same evolution equations as for the one given in the δN formalism, which therefore explicitly establishes the equivalence at the fully nonlinear level between the δN formalism and the covariant formalism.

5 Isocurvature perturbation in the covariant formalism

Finally we will show that the evolution equation (13), if the fluid consists of multiple interacting fluids, can be written in such a way that the right-hand side becomes the sum over all the possible combination of the isocurvature perturbations between two different components, which resembles the standard one obtained under the linear approximation. By setting a coordinate system, we clarify the relation between the isocurvature covector and the isocurvature perturbation at the fully nonlinear level.

To rewrite the right hand side of Eq. (13) in the desired way, we first introduce the curvature covector for the fluid component A by

$$\zeta_a^A \equiv \partial_a N - \frac{\Theta}{3\dot{\rho}^A} \partial_a \rho^A, \quad (23)$$

and define the isocurvature covector S_a^{AB} between the fluids A and B as [25]

$$S_a^{AB} \equiv 3(\zeta_a^A - \zeta_a^B) = -\Theta \left(\frac{\partial_a \rho^A}{\dot{\rho}^A} - \frac{\partial_a \rho^B}{\dot{\rho}^B} \right). \quad (24)$$

We can then rewrite the right-hand side of Eq. (13) as

$$-\frac{\Theta}{3(\rho + P)} \left(\partial_a P - \frac{\dot{P}}{\dot{\rho}} \partial_a \rho \right) = -\frac{\Theta}{3(\rho + P)} \Gamma_a^{(\text{int})} - \frac{\Theta}{6\dot{\rho}^2} \sum_{AB} \left(\frac{\dot{P}^A}{\dot{\rho}^A} - \frac{\dot{P}^B}{\dot{\rho}^B} \right) \dot{\rho}^A \dot{\rho}^B S_a^{AB}, \quad (25)$$

where we have assumed that there is no dissipation in total and

$$\Gamma_a^{(\text{int})} \equiv \sum_A \left(\partial_a P^A - \frac{\dot{P}^A}{\dot{\rho}^A} \partial_a \rho^A \right) \quad (26)$$

is the sum of the intrinsic nonadiabatic perturbations for each fluid. Therefore, Eq. (13) becomes

$$\dot{\zeta}_a = -\frac{\Theta}{3(\rho + P)}\Gamma_a^{(\text{int})} - \frac{\Theta}{6\dot{\rho}^2}\sum_{AB}\left(\frac{\dot{P}^A}{\dot{\rho}^A} - \frac{\dot{P}^B}{\dot{\rho}^B}\right)\dot{\rho}^A\dot{\rho}^BS_a^{AB}. \quad (27)$$

This result is a fully nonlinear generalization of [26] and obtained in Ref. [25]. One can clearly see that the evolution of the curvature covector is generated by the existence of isocurvature covectors and the relative difference of the sound speeds of fluids.

It is important to notice that we can apply this equation even when energy is exchanged among the fluids. To see it in a more manifest way, let us introduce the effect of the energy transfer by

$$\dot{\rho}^A + \Theta(\rho^A + P^A) = Q^A, \quad (28)$$

where Q^A represents a rate of energy density that the fluid A absorbs. Since the energy is conserved in total, we have $\sum_A Q^A = 0$. Using this equation, we find that Eq. (27) can be written as

$$\begin{aligned} \dot{\zeta}_a = & -\frac{\Theta}{6}\sum_{AB}\left(\frac{\dot{P}^A}{\dot{\rho}^A} - \frac{\dot{P}^B}{\dot{\rho}^B}\right)\frac{\rho^A + P^A}{\rho + P}\frac{\rho^B + P^B}{\rho + P}\left(\frac{\partial_a\rho^A}{\rho^A + P^A} - \frac{\partial_a\rho^B}{\rho^B + P^B}\right) \\ & -\frac{\Theta^2}{3\dot{\rho}^2}\left(\dot{\rho}\Gamma_a^{(\text{int})} + \langle Q\rangle\partial_a\rho\right), \end{aligned} \quad (29)$$

where $\langle Q\rangle \equiv \sum_A c_A^2 Q^A$. As is clear, the last term represents a contribution from the energy transfer but the energy transfer also affects the other terms implicitly through the evolution equation (28).

If each fluid obeys a barotropic equation of state [$\Gamma_a^{(\text{int})} = 0$], by applying the coordinate system given by Eq. (1) to Eq. (29), the evolution equation of the curvature perturbation corresponding to Eq. (29) can be derived. Choosing again the uniform energy density hypersurface, the last term of Eq. (29) vanishes and we end up with the following equation:

$$\partial_i\psi' = -\frac{\mathcal{N}\Theta}{6}\sum_{AB}(c_A^2 - c_B^2)\frac{\rho^A + P^A}{\rho + P}\frac{\rho^B + P^B}{\rho + P}\partial_i S^{AB}, \quad (30)$$

where $S^{AB} \equiv 3(\zeta^A - \zeta^B)$ is the fully nonlinear isocurvature perturbation introduced in [27] and ζ^A is defined by

$$\zeta^A(t, x^i) \equiv \psi(t, x^i) + \frac{1}{3}\int_{\bar{\rho}^A(t)}^{\rho^A(t, x^i)}\frac{d\rho^A}{\rho^A + P^A(\rho^A)}. \quad (31)$$

Thus, we have shown that the evolution of the curvature covector in the covariant formalism is equivalent to that of the standard curvature perturbation for multiple interacting fluids as well. Notice that $\dot{P}^A/\dot{\rho}^A$ coincides with the sound velocity squared c_A^2 for a barotropic fluid.

6 Summary

In this paper, we have shown the fully nonlinear equivalence of curvature perturbations on superhorizon scales from the two formalisms: the δN and the covariant formalisms. In particular, by setting a coordinate system and integrating over space, we have identified the evolution equation of the curvature covector with that of the curvature perturbation in the δN formalism. The key assumption here is that the matter energy-momentum tensor takes the perfect fluid form on superhorizon scales, i.e., the continuity equation (12) completely describes the nonlinear evolution of the energy density of the system.

We have also clarified the relation between the isocurvature covector and the nonlinear isocurvature perturbation for multiple interacting fluids. This identification enables us to bridge the results of standard perturbation theory to the covariant formalism. Our treatment here is somewhat less general than [25] but it is enough to obtain the important relation to the standard curvature perturbation. It would be interesting to extend our treatment to systems with multiple scalar, spinor and vector fields, where the next order in gradient expansion may become important.

Acknowledgments

This work was partially supported by the Grant-in-Aid for JSPS Fellows No. 1008477 (T.S.), the TRR 33 “The Dark Universe” (Y.W.) and the Grant-in-Aid for JSPS Scientific Research No. 21740187 (M.Y.).

References

- [1] E. Komatsu *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **192**, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]].
- [2] H. V. Peiris *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **148**, 213 (2003) [astro-ph/0302225].
- [3] J. M. Bardeen, *Phys. Rev. D* **22**, 1882 (1980); H. Kodama and M. Sasaki, *Prog. Theor. Phys. Suppl.* **78**, 1 (1984); V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, *Phys. Rept.* **215**, 203 (1992).
- [4] J. M. Maldacena, *JHEP* **0305**, 013 (2003) [astro-ph/0210603]; V. Acquaviva, N. Bartolo, S. Matarrese and A. Riotto, *Nucl. Phys. B* **667**, 119 (2003) [astro-ph/0209156].
- [5] C. Germani and Y. Watanabe, *JCAP* **1107**, 031 (2011) [arXiv:1106.0502 [astro-ph.CO]].
- [6] D. Seery and J. E. Lidsey, *JCAP* **0506**, 003 (2005) [astro-ph/0503692]; X. Chen, M. -x. Huang, S. Kachru and G. Shiu, *JCAP* **0701**, 002 (2007) [hep-th/0605045];

- S. Mizuno and K. Koyama, Phys. Rev. D **82**, 103518 (2010) [arXiv:1009.0677 [hep-th]]. A. De Felice and S. Tsujikawa, JCAP **1104**, 029 (2011) [arXiv:1103.1172 [astro-ph.CO]]. T. Kobayashi, M. Yamaguchi and J. 'i. Yokoyama, Phys. Rev. D **83**, 103524 (2011) [arXiv:1103.1740 [hep-th]].
- [7] T. Suyama, T. Takahashi, M. Yamaguchi and S. Yokoyama, JCAP **1012**, 030 (2010) [arXiv:1009.1979 [astro-ph.CO]].
- [8] A. D. Linde and V. F. Mukhanov, Phys. Rev. D **56**, 535 (1997) [arXiv:astro-ph/9610219]; K. Enqvist and M. S. Sloth, Nucl. Phys. B **626**, 395 (2002) [arXiv:hep-ph/0109214]; D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002) [arXiv:hep-ph/0110002]; T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001) [Erratum-ibid. B **539**, 303 (2002)] [arXiv:hep-ph/0110096].
- [9] T. Moroi and T. Takahashi, Phys. Rev. D **66**, 063501 (2002) [arXiv:hep-ph/0206026]. D. H. Lyth, C. Ungarelli and D. Wands, Phys. Rev. D **67**, 023503 (2003) [arXiv:astro-ph/0208055]; D. H. Lyth and D. Wands, Phys. Rev. D **68**, 103516 (2003) [arXiv:astro-ph/0306500]; N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D **69**, 043503 (2004) [arXiv:hep-ph/0309033]; K. Ichikawa, T. Suyama, T. Takahashi and M. Yamaguchi, Phys. Rev. D **78**, 023513 (2008) [arXiv:0802.4138 [astro-ph]].
- [10] G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D **69**, 023505 (2004) [arXiv:astro-ph/0303591]; L. Kofman, arXiv:astro-ph/0303614.
- [11] M. Zaldarriaga, Phys. Rev. D **69**, 043508 (2004) [arXiv:astro-ph/0306006]; T. Suyama and M. Yamaguchi, Phys. Rev. D **77**, 023505 (2008) [arXiv:0709.2545 [astro-ph]]; K. Ichikawa, T. Suyama, T. Takahashi and M. Yamaguchi, Phys. Rev. D **78**, 063545 (2008) [arXiv:0807.3988 [astro-ph]].
- [12] A. A. Starobinsky, JETP Lett. **42** (1985) 152 [Pisma Zh. Eksp. Teor. Fiz. **42** (1985) 124]; M. Sasaki and E. D. Stewart, Prog. Theor. Phys. **95**, 71 (1996); [arXiv:astro-ph/9507001]; H. Kodama and T. Hamazaki, Phys. Rev. D **57**, 7177 (1998) [gr-qc/9712045]. M. Sasaki and T. Tanaka, Prog. Theor. Phys. **99**, 763 (1998). [arXiv:gr-qc/9801017]; Y. Nambu and A. Taruya, Class. Quant. Grav. **15**, 2761 (1998) [arXiv:gr-qc/9801021].
- [13] D. Langlois and F. Vernizzi, Phys. Rev. Lett. **95**, 091303 (2005) [astro-ph/0503416]; D. Langlois and F. Vernizzi, Phys. Rev. D **72**, 103501 (2005) [astro-ph/0509078].
- [14] D. Langlois and F. Vernizzi, JCAP **0702**, 017 (2007) [astro-ph/0610064]; S. Renaux-Petel and G. Tasinato, JCAP **0901**, 012 (2009) [arXiv:0810.2405 [hep-th]]; D. Langlois and F. Vernizzi, Class. Quant. Grav. **27**, 124007 (2010) [arXiv:1003.3270 [astro-ph.CO]].

- [15] G. I. Rigopoulos and E. P. S. Shellard, JCAP **0510**, 006 (2005) [astro-ph/0405185]; G. I. Rigopoulos, E. P. S. Shellard and B. J. W. van Tent, Phys. Rev. D **76**, 083512 (2007) [astro-ph/0511041]; E. Tzavara and B. van Tent, JCAP **1106**, 026 (2011) [arXiv:1012.6027 [astro-ph.CO]].
- [16] D. Wands, K. A. Malik, D. H. Lyth and A. R. Liddle, Phys. Rev. D **62**, 043527 (2000) [astro-ph/0003278].
- [17] D. H. Lyth, K. A. Malik, and M. Sasaki, J. Cosmol. Astropart. Phys. **05**, 004 (2005).
- [18] K. Enqvist, J. Hogdahl, S. Nurmi and F. Vernizzi, Phys. Rev. D **75**, 023515 (2007) [gr-qc/0611020].
- [19] J. -L. Lehnars and S. Renaux-Petel, Phys. Rev. D **80**, 063503 (2009) [arXiv:0906.0530 [hep-th]].
- [20] K. A. Malik and D. Wands, Class. Quantum. Grav. **21**, L65 (2004) [astro-ph/0307055]; D. H. Lyth and Y. Rodriguez, Phys. Rev. D **71**, 123508 (2005) [astro-ph/0502578]; K. A. Malik and D. Wands, Phys. Rept. **475**, 1 (2009) [arXiv:0809.4944 [astro-ph]]; A. J. Christopherson and K. A. Malik, JCAP **0911**, 012 (2009) [arXiv:0909.0942 [astro-ph.CO]].
- [21] D. Langlois, F. Vernizzi and D. Wands, JCAP **0812**, 004 (2008) [arXiv:0809.4646 [astro-ph]].
- [22] Y. Watanabe, arXiv:1110.2462 [astro-ph.CO], [Phys. Rev. D (to be published)].
- [23] J. -L. Lehnars and P. J. Steinhardt, Phys. Rev. D **78**, 023506 (2008) [arXiv:0804.1294].
- [24] M. Shibata and M. Sasaki, Phys. Rev. D **60**, 084002 (1999) [arXiv:gr-qc/9905064]. T. Hamazaki, Phys. Rev. D **78**, 103513 (2008) [arXiv:0811.2366 [astro-ph]].
- [25] D. Langlois and F. Vernizzi, JCAP **0602**, 014 (2006) [astro-ph/0601271].
- [26] K. A. Malik, D. Wands and C. Ungarelli, Phys. Rev. D **67**, 063516 (2003) [arXiv:astro-ph/0211602].
- [27] M. Kawasaki, K. Nakayama, T. Sekiguchi, T. Suyama and F. Takahashi, JCAP **0811**, 019 (2008) [arXiv:0808.0009 [astro-ph]].